Solitons with Compact Support in Multi-dimensional Nonlinear Wave Equations

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Solitons are nonlinear traveling waves where the nonlinearity and dispersion are balanced to create a stable coherent local environment so that that the solitions maintain their coherence when colliding with other solitions. Because dispersive is much stronger to two and three dimensions than in one dimension, this balance is usually lost when a one- dimensional equation is generalized to higher dimensions. Until recently, most known solitions were one-dimensional waves, even when they were observed as coherent waves in multidimensional equations. I will describe a new class of fully two and three dimensional solitons where the balance is maintained by a nonlinear dispersion term that weakens the dispersion near the edge of the solition. These solitons are cylindrically symmetric in two dimensions and spherically symmetric in three dimensions. Also, the nonlinear dispersion decays quickly at the edge of the solitons and the balance of forces compactifies these traveling wave solutions creating a compact solition, called a compacton, that is exactly zero outside of its central core region.

For the computed solutions of these third-order partial differential equations to be accurate, the numerical method must preserve a delicate balance between the nonlinear dispersion and convection forces. This balance is easily lost in numerical approximation if the aliasing, due to the nonlinearities, is not handled carefully. I will describe how we preserved this balance by combination of using a pseudo-spectral method to approximate the spatial derivatives, an artificial hyper-viscosity to account for the loss of smoothness at the edge of the compacton, and a time filtering method to separate the slow time scales of the compaction motion from the exceptionally fast time-scales of the nonlinear dispersion.

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